Introduction to Turbulence Modelling

Miquel Altadill Llasat

Master Thesis

Supervisors:
M.Sc Charalampos Alexopoulos, ITLR
Prof. Dr.-Ing. B. Weigand, ITLR
Prof. Bassam A. Younis Ph.D., UCD

Universität Stuttgart
Institut für Thermodynamik der Luft- und Raumfahrt

ITLR

UCDAVIS
UNIVERSITY OF CALIFORNIA

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Introduction

Turbulent flows are characterised by:

- **Higly unstedy**
- **Three dimensional behabiour**
- **Contain great vorticity**
- **Considerable flow mixing**
- **Contain coherent structures**
- **Length and time scale fluctuations**
- **Considered as random and chaotic**

Chaos and Nonlinearity

But, what does chaos mean?

Chaos theory concerns about deterministic systems whose behaviour can in principle be predicted, but they are highly sensitive to initial conditions (**Butterfly Effect**).

- A chaotic system **must be nonlinear**

The Navier-Stokes equations are a clear example of nonlinearity.

$$\frac{d\rho}{dt} + \nabla \cdot (\rho \vec{v}) = 0 \tag{1.1}$$

$$\frac{d}{dt}(\rho \vec{v}) + \nabla \cdot (\rho \vec{v}\,\vec{v}) = -\nabla p + \nabla \cdot (\vec{\tau}) + \rho \vec{g} \tag{1.2}$$

$$\frac{d}{dt}(\rho(u + e_c)) + \nabla \cdot ((u + e_c)\rho \vec{v}) = -\nabla \cdot (\rho \vec{v}) + \nabla \hat{u}(\vec{v} \cdot \vec{\tau}) - \nabla \cdot \vec{q} + \rho \vec{g} \cdot \vec{v} + G \tag{1.3}$$

The Energy Cascade

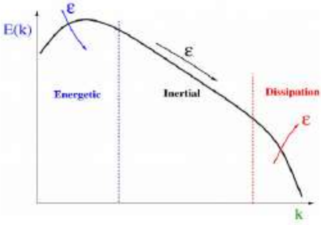
The **energy cascade** concept was introduced by Richardson (1922). The cascade can be divided into three regions as shown in the picture:

- **Energetic:** Kinetic energy enters the turbulence at the largest scales.
- **Inertial:** The energy is transferred (by inviscid processes) to smaller scales.
- **Dissipation:** The energy is dissipated by viscous action.

The term ε is known as **energy dissipation**. The energy spectrum $E(k)$ was apporixmated by Komogorov (K41) as:

$$E(k) = C \cdot \varepsilon^{2/3} k^{-5/3} \quad (1)$$

Where C is a universal constant. This approximation considers the turbulence homogeneously.

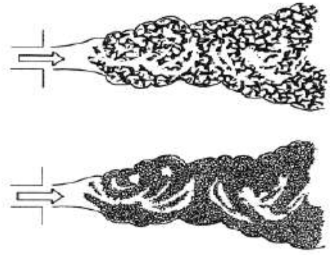


[CTTC. Lecture notes of Computational Engineering. ESEIAAT, 2014]s

The non-linear term in the N-S equations is the **convective term** $\Delta \cdot (\rho \vec{v}\vec{v})$ from the momentum equation in charge of the **energy transport** from the large scales to the small ones. Richardson succintly summarized the matter thus:

Big whorls have little whorls,
Which feed on their velocity;
And little whorls have lesser whorls,
And so on to viscosity
(in the molecular sense).

Turbulence Scales



[CTTC. Lecture notes of Computational Engineering. ESEIAAT, 2014]

The eddies are defined by its length (l), velocity u and timescale τ. The nomenclature:

- **Largest scales:** Eddy(l_0, u_0, τ_0)
- **Smallest scales:** Eddy($l_\eta, u_\eta, \tau_\eta$)

The eddies have an energy order of u_0^2 and the timescale can be expressed as $\tau_0 = l_0/u_0$ so that the ratio of transfer of energy scale as:

$$\frac{u_0^2}{\tau_0} = u_0^3/l_0 \qquad (2)$$

The energy dissipation ε scales as u_0^3/l_0 independent of the kinematic viscosity ν. The Reynolds number is defined by the largest eddies as:

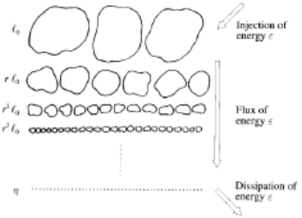
$$R \sim \frac{l_0 u_0}{\nu} \qquad (3)$$

Kolmogorov's Hypothesis

Kolmogorov's hypothesis of local isotropy. *At sufficient high Reynolds number, the small-scale turbulent motions ($l << l_0$) are statistically isotropic.*
In general, the large eddies are anisotropic and are affected by the boundary conditions of the flow.

Kolmogorov's first similarity hypothesis. *In every turbulent flow at sufficiently high Re, the statistics of the small-scale motions have a universal form that is unniquely determined by ν and ε.*

[U. Frisch,Turbulence. The legacy of A.N.Kolmogrov. Cambridge University Press, 1995.]



Kolmogrov Scales

$$\eta \sim \left(\frac{\nu^3}{\varepsilon}\right)^{1/4} \quad (4a)$$

$$u_\eta \sim (\nu\varepsilon)^{1/4} \quad (4b)$$

$$\tau_\eta \sim \left(\frac{\nu}{\varepsilon}\right)^{1/2} \quad (4c)$$

Ratio between large and small scales

$$\eta / l_0 \sim Re^{-3/4} \quad (5a)$$

$$u_\eta / u_0 \sim Re^{-1/4} \quad (5b)$$

$$\tau_\eta / \tau_0 \sim Re^{-1/2} \quad (5c)$$

Turbulence Computational Methods

- **Method 1:** Involves the use of correlations such as ones that give the friction factor as a function of the Re or the Nu of heat transfer as a function of the Re and Pr numbers. Just for simple flow cases.

- **Method 2:** Uses integral equations which can be derived from the equations of motion by integrating over one or more coordinates. Solves simles ODEs.

- **Method 3:** Based on equations obtained by decomposing the equations of motion into mean fluctuationg components. The decomposed equations do not form closed sets so these methods require from **turbulence models**. RANS and LES are inside this group.

- **Method 4**: The Navier-Stokes equations are solved for all the motions in a turbulent flow, well known as DNS.

Only Method 2 and 3 would be briefly discussed.

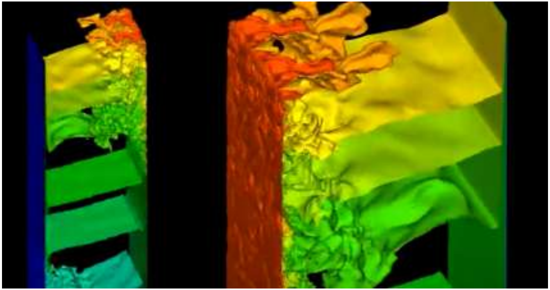
Direct Numerical Simulation (DNS)

Characteristics

- Most accurate approach.
- Solves N-S without averaging or approximation.
- Captures all the significant structures of the flow.
- High computational requirements (memory & speed).
- Grid accurate enough (Kolmogrof scale, η) and limited to simple geometries.

Some **examples** of kinds of uses of DNS are:

- Understanding the process for laminar turbulent transition, as well as the mechanisms of turbulence production, energy transfer and dissipation in turbulent flows;
- Simulation of the production of aerodynamic noise;
- Understanding the **effects of compressibility** on turbulence;
- Understanding the interaction between combustion and turbulence;
- Controlling and reducing drag on a solid surface.



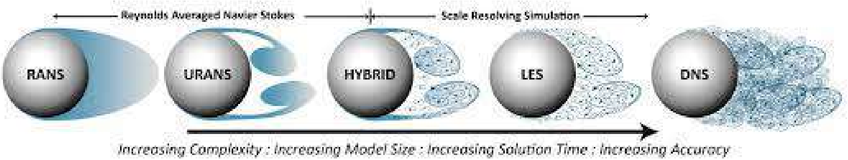
DNS of a Differentially Heated Cavity at Ra=1e11 [F.Xavier Trias]

Reynolds Averaged Navier Stokes (RANS)

Averaging **Types**:

- Averaging over all time → Called **RANS**
- Averaging over ensambles → Called **URANS**

In RANS, the mean flow defined by the average is steady. In URANS the averaging removes all turbulent (random) eddies. Wyngaard (2010) reminds us "that the ensemble-averaged field is unlikely to exist in any realization of a turbulent flow, even for an instant."



[John Hart. Comparison of turbulence modeling approaches to the simulation of a dimpled sphere. ISEA 2016]

Reynolds Averaged Navier Stokes (RANS)

In a statistically steady flow every variable can be written as:

$$\phi(x_i, t) = \overline{\phi}(x_i) + \phi'(x_i, t)\,, \qquad (10.30)$$

where

$$\overline{\phi}(x_i) = \lim_{T \to \infty} \frac{1}{T} \int_0^T \phi(x_i, t)\, \mathrm{d}t\,. \qquad (10.31)$$

$$\frac{\partial(\rho\overline{\phi})}{\partial t} + \frac{\partial}{\partial x_j}\left(\rho \overline{u}_j \overline{\phi} + \rho \overline{u'_j \phi'}\right) = \frac{\partial}{\partial x_j}\left(\Gamma \frac{\partial \overline{\phi}}{\partial x_j}\right)\,. \qquad (10.37)$$

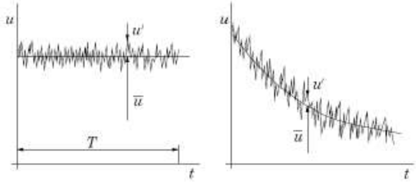


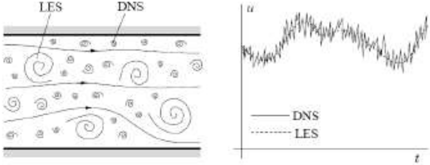
Fig. 10.24 Time-averaging for a statistically steady flow (left) and ensemble-averaging for an unsteady flow (right)

[Ferziger, Peric. Computational Methods for Fluid Dynamics. Springer 4th Ed.]

Large Eddy Simulation (LES)

The Navier-Stokes and scalar equations are filtered (averaged) over space.

It is important to note that while most of the turbulence energy is in the subfilter scale for URANS, LES resolves most of the turbulence energy.

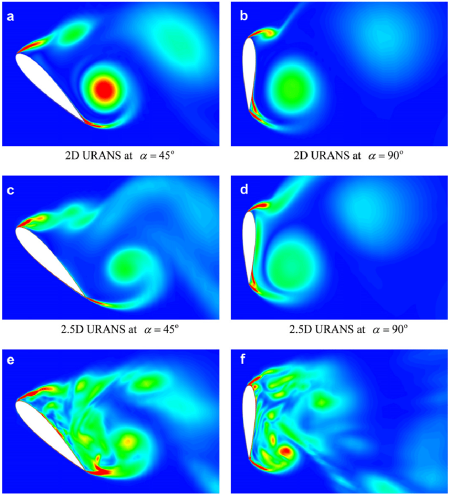


[Ferziger, Peric. Computational Methods for Fluid Dynamics. Springer 4th Ed.]

Momentum equation solved in LES:

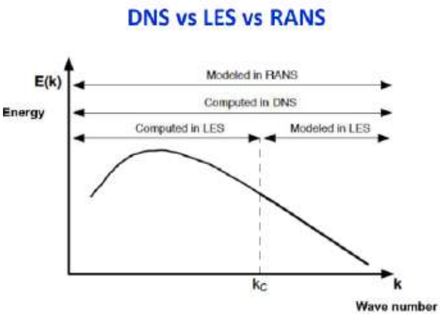
$$\frac{\partial(\rho\overline{u}_i)}{\partial t} + \frac{\partial(\rho\overline{u_i}\overline{u}_j)}{\partial x_j} = \frac{\partial\tau_{ij}^s}{\partial x_j} - \frac{\partial\overline{p}}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\mu\left(\frac{\partial\overline{u}_i}{\partial x_j} + \frac{\partial\overline{u}_j}{\partial x_i}\right)\right] . \qquad (10.12)$$

Comparison



2D URANS at $\alpha = 45^\circ$ | 2D URANS at $\alpha = 90^\circ$
2.5D URANS at $\alpha = 45^\circ$ | 2.5D URANS at $\alpha = 90^\circ$
2.5D LES at $\alpha = 45^\circ$ | 2.5D LES at $\alpha = 90^\circ$

[Chao Li, 2.5D large eddy simulation of vertical axis wind turbine in consideration of high angle of attack flow. 2012]



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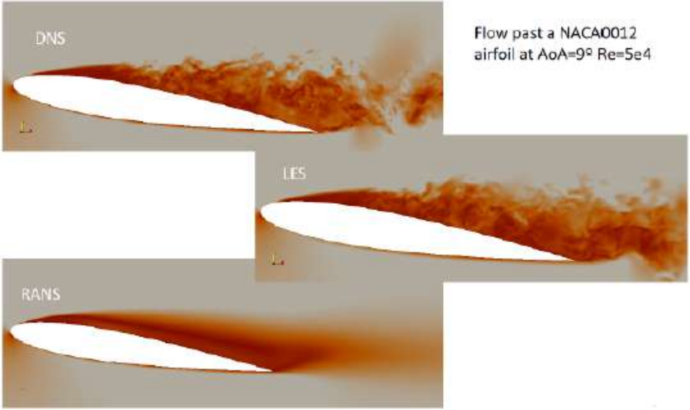

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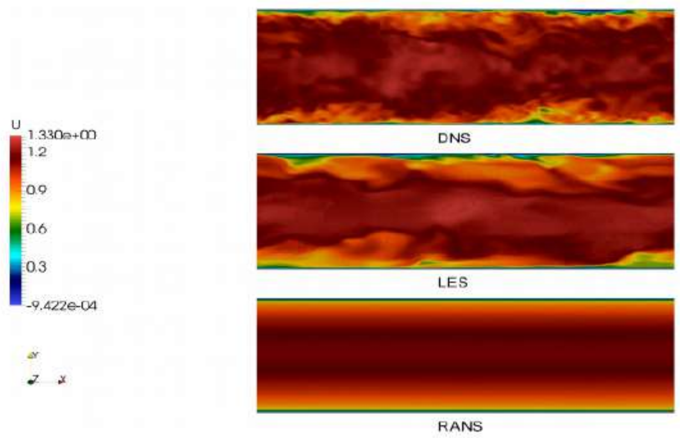

[CTTC. Lecture notes of Computational Engineering. ESEIAAT, 2014]